Feedback Control
Lab-Exercises

Introduction to Information Science and Technology

Prof. Boris Houska

Welcome!

Welcome to the control lab of ShanghaiTech's Introduction to Information Science and Technology course. In this exercise we will learn about the basic concepts of feedback control and its applications. The goal of this course is to learn how to control a car by using a PID controller. You will also learn how to simulate closed-loop systems, visualize, and present your results.

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TIME:

- May 6th, 8:00 - 14:00.

LOCATION:

- SIST, 1A-200

PREPARATION:

- Laptop with Python installed, please charge your Laptop before the lecture, as there may be not enough power plugs in the room.

TOPIC:

- This exercise is about the closed-loop control of a car. Here, the position of the car is modeled by two states, $x(t)$ and $y(t)$, denoting the car's position on the $x - y$-plane at time t. Moreover, the angle $\theta(t)$ denotes the orientation of the car, whose time derivative can be controlled by turning the steering wheel. The dynamic model equations of the car are given by

$$
\begin{aligned}
\dot{x}(t) &= v\cos(\theta(t)) \\
\dot{y}(t) &= v\sin(\theta(t)) \\
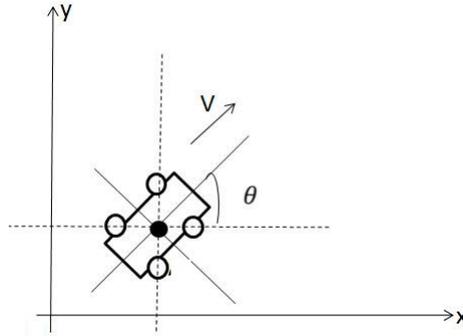\dot{\theta}(t) &= u(t) \, .
\end{aligned} \tag{1}
$$

Figure 1: Visualization of the car model

Notice that the car has a constant velocity of $v = 80 \frac{\text{km}}{\text{h}}$. The goal of this control exercise is to design a PID feedback control law of the form

$$u(t) = K_{\text{P}} e(t) + K_{\text{I}} \int_0^t e(\tau)\, \text{d}\tau + K_{\text{D}} \frac{\partial e(t)}{\partial t} \ ,$$

where

$$e(t) = \left(\begin{array}{c} x(t) - x_{\text{ref}}(t) \\ y(t) - y_{\text{ref}}(t) \end{array} \right) \in \mathbb{R}^2$$

denotes the difference of the car's position at time t to the reference trajectory $(x_{\text{ref}}(t), y_{\text{ref}}(t))$. You may asssume that $e(t)$ can be measured by using a camera.

TASKS:

- In order to get started: download the control lab simulation code example from the SI100 course website and try to run it with Python!

- The first task is to find control gains $K_{\text{P}}, K_{\text{I}}$, and K_{D} such that the car follows the x-axis. Start your car at the point $(x(0), y(0)) = (0\,\text{m}, 10\text{m})$, i.e., not on the x-axis and simulate the closed loop system dynamics. Tune the PID control co-efficients until the car goes in a reasonable time to the x-axis, does not overshoot too much, and follows the line as wanted. **(5 points)**

- Repeat the first exercise, but now make the car go to and move on a circle with a radius of $r = 100\,\text{m}$. You may use the functions

$$\begin{aligned} x_{\text{ref}}(t) &= r \sin\left(\tfrac{vt}{r}\right) \\ \text{and} \qquad y_{\text{ref}}(t) &= r \cos\left(\tfrac{vt}{r}\right) \end{aligned}$$

as a reference trajectory. Another option is to use the function

$$e(t) = x(t)^2 + y(t)^2 - r^2$$

as the error function.   **(5 points)**

- Write a tool in Python (or any other programming language of your choice) that visualizes and animates the car moving to and on the circle. If you like, you can also try to follow more complicated race-tracks of your own choice. There are plenty of online tutorials around, e.g,

https:
//nickcharlton.net/posts/drawing-animating-shapes-matplotlib.html

or similar websites can be used as a starting point.   **(10 points)**

- If you have finished the animation video, present your results to the whole class in a 1 minute presentation. Keep it short and try to explain your main results in 3-4 sentences.   **(10 points)**